

Math 323 - Formal Mathematical Reasoning and Writing
Problem Session
Wednesday, 3/4/15

1. Write each of the following. These are the rules you'll use on the rest of the worksheet.
 - (a) The NEGATION of the statement "if A then B ".
 - (b) The NEGATION of the statement " A AND B ".
 - (c) The NEGATION of the statement " A OR B ".
 - (d) The NEGATION of the statement "for all objects with property P , X happens".
 - (e) The NEGATION of the statement "there exists an object with property P such that X happens".
 - (f) The CONVERSE of the statement $A \Rightarrow B$:
 - (g) The CONTRAPOSITIVE of the statement $A \Rightarrow B$:

2. ¹ Write the negation of each of the following statements. State your answer in such a way that the words “no” and “not” do not appear.

(a) There is a real number x between -1 and 1 such that for all real numbers y between -1 and 1 , $x^2 + y^2 \leq 1$.

(b) For every integer $n \geq 2$, $n^2 \geq 2^n$.

3. Consider the statement: ‘if a is a natural number with a^2 even, then a is even’.

(a) Write the NEGATION of the statement.

(b) Write the CONVERSE of the statement.

(c) Write the CONTRAPOSITIVE of the statement.

(d) Which of the three statements you wrote above are logically equivalent to the original statement?

¹Most of the problems in this worksheet are adapted from *How to Read and Do Proofs* by Daniel Solow, Chapter 8

4. Consider the statement: 'if S is a set of real numbers that is not bounded above, then for every $r \in \mathbb{R}$ there exists an $s \in S$ such that $r < s$.'
- (a) Write the NEGATION of the statement.

 - (b) Write the CONVERSE of the statement.

 - (c) Write the CONTRAPOSITIVE of the statement.

 - (d) Which of the three statements you wrote above are logically equivalent to the original statement?
5. Let m, n be positive integers. Consider the statement: 'if $\gcd(m, n)$ is odd, then m or n is odd.'
- (a) Write the NEGATION of the statement.

 - (b) Write the CONVERSE of the statement.

 - (c) Write the CONTRAPOSITIVE of the statement.

 - (d) Which of the three statements you wrote above are logically equivalent to the original statement?

L^AT_EX tip of the week!

Notice that you can't just type `\` into T_EX, since this symbol is already taken by the language. You can use the command `\backslash`, but this will make spacing weird in mathematics. For the correct spacing, use `\setminus`:

`A \setminus B` gives us $A \setminus B$

Here are a few other symbol variants you might want to use. The examples given here are usually interchangeable, but you may prefer one to the other for aesthetic reasons. Never use both variants in a single proof, though! Pick one, and stick with it.

<code>\epsilon</code>	ϵ	<code>\varepsilon</code>	ε
<code>\phi</code>	ϕ	<code>\varphi</code>	φ
<code>\emptyset</code>	\emptyset	<code>\varnothing</code>	\varnothing